Government Financing, Inflation, and the Financial Sector[*]

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Abstract

We calculate the effects of an increase in government spending financed with labor income taxes or inflation. Government spending takes the form of government consumption or transfers. In the model, agents increase the use of financial services to avoid losses from inflation, as empirically the financial sector increases with inflation. In standard cash-in-advance models, the financial sector size is constant, which implies that it is optimal to finance the government with inflation. In our framework, it is optimal to use taxes rather than inflation. This result is robust to alternative specifications and definitions of seigniorage and government spending.

JEL Codes: E52, E62, E63

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1 Introduction

We calculate the effects of an increase in government spending in a model in which the changes in the behavior of agents toward inflation are reflected in the size of the financial sector. We consider government spending in the form of government consumption or transfers. It is important to consider transfers as this form of government spending increased substantially in the last decades. Moreover, it has been shown that the existence of transfers implies optimal positive inflation in cash-in-advance models (Adão and Silva 2019).[1] In the model, government spending can be financed with inflation or distortionary taxes in the form of labor income taxes. Agents choose the frequency of trades of bonds for money. Standard cash-in-advance models have constant frequency of trades and constant size of the financial sector. We let agents decide the frequency of trades. This change implies different predictions on the effects of financing the government with inflation.

When inflation increases, agents increase the use of financial services to decrease money balances. When the costs implied by the increase in the use of financial services are taken into account, the welfare cost of inflation increases. As a result, it becomes optimal to avoid inflation and finance the government with taxes. Inflation might be optimal at first sight, as transfers imply optimal positive inflation in cash-in-advance models. However, once the additional costs of inflation are taken into account, the objective of inflation is obtained more effectively with labor taxes.[2] It is important to consider changes in the trading frequency to calculate the effects of policies that involve changes in inflation.

Consider the results from our simulations of an increase in government spending in the form of transfers that corresponds to an increase from 20 to 21 percent of the government-output ratio. If the increase in spending is financed with an increase in labor income taxes then having fixed or endogenous trading frequency implies similar predictions for the welfare cost: 0.95 percent in terms of income. If the increase in spending is financed with an increase in inflation, the predicted welfare cost with fixed trading frequency is 0.49 percent in terms of

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[1]Positive transfers are optimal, for example, with heterogeneous agents when the weight on poor households is high or when inequality is high (Werning 2007). From 1947 to 2016, government expenditures increased from 16.5% to 17.6% of GDP and total government transfer payments increased from 4.6% to 15% of GDP (data from the St. Louis Fed FRED dataset).

[2]Related to this result, Von Mises (1981, p. 262) states that "all the aims of inflationism can be secured by other sorts of intervention in economic affairs, and secured better, and without undesirable incidental effects."

income. With endogenous periods, the predicted welfare cost is 1.45 percent. A difference of 0.96 percentage points.[3]

One of the reasons for the difference in estimates is the larger decrease in the demand for money with endogenous trading frequency when inflation increases. The decrease in the demand for money implies higher costs for the provision of financial services. Moreover, for the same increase in the inflation rate, the decrease in the demand for money implies that the increase in seigniorage is smaller with endogenous trading frequency. With fixed trading frequency, the inflation rate necessary to cover the increase in government spending is equal to 5.5 percent per year. In contrast, with endogenous periods, the inflation rate is equal to 12.7 percent per year. A model with fixed trading frequency predicts that a smaller inflation rate generates the same seigniorage as a higher inflation rate in a model with endogenous trading frequency.

The values for inflation and seigniorage implied by this example are within realistic estimates. According to our simulations, seigniorage revenues as a percentage of output in this example are 2.19 percent of output with fixed trading frequency and 1.93 percent of output with endogenous trading frequency. Sargent et al. (2009) estimate that seigniorage exceeded 10 percent of output in Brazil in 1993 and in Argentina in several periods during 1970-1990. Click (1998) calculates seigniorage to be 2.5 percent of output on average in a database of 90 countries during 1971-1990 and finds that seigniorage exceeded 10 percent of output in some countries.[4] Kimbrough (2006) cites evidence that seigniorage may reach 17 percent of GDP in some countries but that usually it does not exceed 10 percent. Kimbrough, to analyze revenues generated from inflation, considers seigniorage from 5 to 15 percent of output. Our simulations imply an increase in seigniorage revenues of about 1 percentage point, and total seigniorage revenues around 2 percent of output, which is much below the admitted maximum value of 10 percent.

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[3]For an idea of the magnitude, one percent of income is equivalent to more than 130 billion dollars every year or more than one thousand dollars distributed to every household in the United States every year (2010 dollars, data from the BEA and from the US Census Bureau).

[4]Click (1998) calculates seigniorage to be 10.5 percent of government spending on average but to exceed 100 percent of government spending for various countries. In this example, seigniorage is 10.3 percent of government consumption with fixed trading frequency and 9.2 percent with endogenous trading frequency. In the database of Click, U.S. has a seigniorage of 0.43 percent of output. The lowest seigniorage is 0.38 percent of output, for New Zealand. Our model reproduces these values with low inflation. The highest seigniorage in the database is 14.8 percent of output, for Israel.

Having endogenous or fixed trading frequency is not relevant when the increase in spending is financed by an increase in labor income taxes. When financing is made through labor income taxes, inflation is the same before and after the increase in spending. Changes in the demand for money are then insignificant. The changes in labor taxes are approximately the same with endogenous or fixed intervals; labor taxes increase from 29.8 to 32.1 percent. Other effects, such as those on output and consumption, are also similar when financing is made through taxes. Endogenous trading frequency is relevant when the change in policy implies a change in inflation and, consequently, a significant change in the demand for money.[5]

Agents in the model pay a fee in goods to have access to financial markets, where they trade bonds for money. The payment of a fee implies staggered visits to the asset market. Agents are heterogeneous, as the moments in which they trade bonds for money are different, and their bond and money holdings are different. As inflation increases, it is optimal to increase the frequency of the trades in the financial markets to decrease real money holdings. The increase in the frequency of trades increases the frequency of fee payments.

We interpret the aggregate fee payments in the model as the size of the financial sector. We can in this way use our model to estimate the increase in the financial sector implied by the increase in inflation. In our simulations, an increase in inflation from 0 to 10 percent per year implies an increase in financial services of 1 percentage point. Our estimates are in accordance with the evidence in English (1999). English finds a positive relation between inflation and the size of the financial sector and estimates that a 10 percent increase in inflation in U.S. implies an increase in the financial sector of 1.3 percentage points.

Cooley and Hansen (1991, 1992) study the effects of inflation in a cash-in-advance economy with distortionary taxation. Their economy is similar to the economy that we have here, except that we allow agents to choose optimally the timing of their transactions in the asset market. In Cooley and Hansen (1991), decreasing inflation from 10 to zero percent per year, with seigniorage revenues replaced with labor income taxes, implies a welfare loss of 1.018 percent of output. We emphasize that this exercise generates a welfare loss in Cooley and Hansen (1991) instead of a welfare gain.[6] In a different study, Cooley and Hansen (1992)

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[5]The area under the demand curve does not approximate the welfare cost of inflation here. The welfare cost is higher than the area under the demand for money with either endogenous or fixed trading frequency. The reason is the existence of distortionary taxation, government spending, and endogenous labor supply.

[6]About this unexpected result, see also Benabou (1991) and Wright (1991).

find that financing the government with distortionary taxes implies a welfare cost of 13.30 percent of GDP when compared with lump sum taxation. If financing is done with inflation, the welfare cost is smaller, 12.43 percent. The difference of $13.30 - 12.43 = 0.87$ percent is the loss of financing the government with distortionary taxes instead of inflation. We confirm that replacing inflation with labor income taxes implies a welfare loss when agents cannot choose the frequency of transactions in the asset market. Here, once we lift the restriction on the frequency of asset transactions, the loss of decreasing inflation is reversed to a gain.

Silva (2012) introduces a model to study trading frequency and the welfare cost of inflation, assuming no government spending and that lump sum taxation is available. Here, we introduce distortionary taxation and government spending in the form of government consumption or government transfers. We also consider different forms in which the government can finance itself. We find that taking into account the decision on the frequency of trades substantially changes the estimates of the welfare cost of inflation under distortionary taxation. This effect is so large in certain cases that it reverses a prescription of increasing inflation to a prescription of keeping inflation low.

We also find that the response of output depends on the trading frequency being considered. With fixed trading frequency, inflation acts as a labor income tax. An increase in inflation encourages agents to substitute away from labor toward leisure, which decreases output. This effect is also present in the endogenous trading frequency case. However, there is an additional effect. As the opportunity cost of money increases agents increase their demand for financial services. As the financial sector requires labor and capital, the increase in the financial sector implies a positive effect on labor demand. The effect on labor hours is small as the financial sector as a fraction of output increases only one percentage point when the increase in spending is financed through inflation. However, the effect might be enough to change the sign of the output response. Under certain conditions, there is a positive government multiplier. If the increase in spending is in government consumption, the multiplier is -0.01 with fixed timing and 1.09 with endogenous timing.[7]

The remainder of the paper is organized as follows. Section 2 describes the model. In

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[7]According to Ramey (2011), the evidence points to a government spending multiplier between 0.8 and 1.5. For permanent increases in government spending, the multiplier is closer to the one in our experiments. In this case, Ramey points to a multiplier of 1.2. See also Hall (2009) and Woodford (2011).

section 3, we solve the model for the competitive equilibrium steady state. In section 4, we specify the welfare cost criterion, discuss our simulations, and report the results of an increase in government spending financed by distortionary taxes or inflation. Section 5 introduces the decision between cash and credit goods. Section 6 concludes.

2   The Model

We introduce distortionary taxation, government consumption, and government transfers to a model in which agents exchange bonds for money infrequently and decide their frequency of trading. These modifications allow us to study how the government affects the economy and financial markets if it increases expenditures in the form of transfers or government consumption, and finances the increase in expenditures in different ways.

Money must be used to purchase goods, only bonds receive interest payments, and there is a cost to transfer money from bond sales to the goods market, as in Baumol (1952) and Tobin (1956). Different from Alvarez et al. (2009) and other models in which there is a separation between the asset market and the goods market (e.g. Grossman and Weiss 1983, Rotemberg 1984), the timing of the infrequent sales of bonds for money is endogenous. Moreover, we have capital and labor decisions, government spending, and distortionary taxation.[8]

Time is continuous and denoted by $t \in [0, +\infty)$. At any moment, there are markets for assets, for the consumption good, and for labor. There are three assets, money, claims to physical capital, and nominal bonds. The markets for assets and the market for the consumption good are physically separated.

There is an unit mass of infinitely lived agents with preferences over consumption and leisure. Agents have two financial accounts: a brokerage account and a bank account. They hold assets in the brokerage account and money in the bank account. We assume that readjustments in the brokerage account have a fixed cost. As only money can be used to buy goods, agents need to maintain an inventory of money in their bank account large enough to pay for their flow of consumption expenditures until the next transfer of funds.

Let M_0 denote money in the bank account at time zero. Let B_0 denote nominal government

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[8]The key differences from standard cash-in-advance models such as Cooley and Hansen (1989) are the heterogeneity of agents and the decision on the trading frequency.

bonds and k_0 claims to physical capital, both in the brokerage account at time zero. Index agents by $s = (M_0, B_0, k_0)$.

The agents pay a cost Γ in goods to transfer resources between the brokerage account and the bank account. Γ represents a fixed cost of portfolio adjustment. Firms in the financial sector are perfectly competitive and obtain zero profits. Let $T_j(s)$, $j = 1, 2, ..$, denote the times of the transfers of agent s. Let $P(t)$ denote the price level. At $t = T_j(s)$, agent s pays $P(T_j(s))\Gamma$ to make a transfer between the brokerage account and the bank account. The agents choose the times $T_j(s)$ of the transfers.

The consumption good is produced by firms. Firms hire labor and rent capital to produce the good. The production function is given by $Y(t) = Y_0 K(t)^\theta H(t)^{1-\theta}$, where $0 < \theta < 1$ and $K(t)$ and $H(t)$ are the aggregate quantities of capital and hours of work at time t. Firms are perfectly competitive. Capital depreciates at the rate δ, $0 < \delta < 1$.

The agent is a composition of a shopper, a trader, and a worker. The shopper uses money in the bank account to buy goods, the trader manages the brokerage account, and the worker supplies labor to the firms. The firms transfer their sales proceeds to their brokerage accounts and convert them into bonds.[9]

The firms pay $w(t)h(t, s)$ and $r^k(t)k(t, s)$ to the worker for the hours of work $h(t, s)$ and capital $k(t, s)$; $w(t)$ are real wages and $r^k(t)$ is the real interest rate on capital. The firms make the payments with a transfer from the brokerage account of the firm to the brokerage account of the agent. When the firms make the payments to the agents, the government collects $\tau_L w(t)h(t, s)$ in labor income taxes and sends T in transfers to the agents. As a result. the brokerage account of the worker is credited by $(1 - \tau_L)w(t)h(t, s) + r^k(t)k(t, s) + T$. These credits can be used at the same date for purchases of bonds.

The government issues bonds that pay a nominal interest rate $r(t)$. Let the price of a bond at time zero be given by $Q(t)$, with $Q(0) = 1$. The nominal interest rate is $r(t) \equiv -d \log Q(t)/dt$. Let inflation be denoted by $\pi(t)$, $\pi(t) = d \log P(t)/dt$. To avoid the opportunity of arbitrage between bonds and capital, the nominal interest rate and the payment of claims to capital satisfy $r(t) - \pi(t) = r^k(t) - \delta$. That is, the rate of return on bonds must be equal to the

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[9]In Silva (2012), firms keep a fraction a of the sales proceeds in money and transfer the remaining fraction $1 - a$ to their brokerage accounts of the workers, $0 \le a < 1$. However, the value of a has little impact on the welfare cost, on the demand for money and on other equilibrium values.

real return on physical capital discounted by depreciation. With this condition satisfied, the agents are indifferent between converting their income into bonds or capital.

Money holdings at time t of agent s are denoted by $M(t,s)$. Money holdings just after a transfer are denoted by $M^+(T_j(s),s)$ and they are equal to $\lim_{t \to T_j, t > T_j} M(t,s)$. Analogously, $M^-(T_j(s),s) = \lim_{t \to T_j, t < T_j} M(t,s)$ denotes money just before a transfer. The net transfer from the brokerage account to the bank account is given by $M^+ - M^-$. If $M^+ < M^-$, the agent makes a negative net transfer, a transfer from the bank account to the brokerage account, which is immediately converted into bonds. Money holdings in the brokerage account are zero, as bonds receive interest payments and it is not possible to buy goods directly with money in the brokerage account. All money holdings are in the bank account. To have M^+ just after a transfer at $T_j(s)$, agent s needs to transfer $M^+ - M^- + P(T_j(s))\Gamma$ to the bank account, $P(T_j(s))\Gamma$ is used to buy goods to pay the transfer cost.

Define a holding period as the interval between two consecutive transfer times, that is $[T_j(s), T_{j+1}(s))$. The first time agent s adjusts its portfolio of bonds is $T_1(s)$ and the first holding period of agent s is $[0, T_1(s))$. To simplify the exposition, let $T_0(s) \equiv 0$, but there is not a transfer at $t = 0$, unless $T_1(s) = 0$.

Denote $B^-(T_j(s),s)$, $B^+(T_j(s),s)$, $k^-(T_j(s),s)$, and $k^+(T_j(s),s)$ the quantities of bonds and capital just before and just after a transfer. During a holding period, bond holdings and capital holdings of agent s follow

$$\dot{B}(t,s) = r(t)B(t,s) + P(t)(1-\tau_L)w(t)h(t,s) + T, \tag{1}$$

$$\dot{k}(t,s) = (r^k(t) - \delta)k(t,s). \tag{2}$$

The way in which equations (1) and (2) are written implies that labor income and government transfers are converted into nominal bonds, and that interest payments to capital are converted into new claims to capital. This is done to simplify the expressions of the law of motion of bonds and capital. The agent is indifferent between the asset allocations on bonds and capital, as $r(t) - \pi(t) = r^k(t) - \delta$.

At each time $T_j(s)$, $j = 1, 2, \ldots$, agent s readjusts the portfolio. At time $T_j(s)$, individual

holdings of money, bonds, and capital satisfy

$$M^+(T_j) + B^+(T_j) + P(T_j)k^+(T_j) + P(T_j)\Gamma = M^-(T_j) + B^-(T_j) + P(T_j)k^-(T_j), \quad (3)$$

for $j = 1, 2, \ldots$ That is, money, bonds and capital holdings just after the readjustment, plus the readjustment cost, must be equal to the previous holdings of money, bonds and capital. With the evolution of bonds and capital in (1)-(2), we can write $B^-(T_j)$ and $k^-(T_j)$ as a function of the interest payments accrued during a holding period $[T_{j-1}, T_j)$. Substituting recursively and using the non-Ponzi conditions $\lim_{j \to +\infty} Q(T_j)B^+(T_j) = 0$ and $\lim_{j \to +\infty} Q(T_j)P(T_j)k^+(T_j) = 0$, we obtain the present value constraint

$$\sum_{j=1}^{\infty} Q(T_j(s)) \left[M^+(T_j(s), s) + P(T_j)\Gamma \right] \leq \sum_{j=1}^{\infty} Q(T_j(s))M^-(T_j) + W_0(s), \quad (4)$$

where $W_0(s) = B_0 + P_0 k_0 + \int_0^\infty Q(t)P(t)(1 - \tau_L)w(t)h(t,s)dt$. Equation (4) states that the present value of money transfers and transfer fees is equal to the present value of deposits in the brokerage account, plus the initial bond and capital holdings.

In addition to the present value budget constraint (4), the agents face a cash-in-advance constraint

$$\dot{M}(t,s) = -P(t)c(t,s), t \geq 0, t \neq T_1(s), T_2(s), \ldots \quad (5)$$

This constraint emphasizes the transactions role of money, that is, agents need money to buy goods. At $t = T_1(s), T_2(s), \ldots$, constraint (5) is replaced by $\dot{M}(T_j(s), s)^+ = -P(T_j(s))c^+(T_j(s))$, where $\dot{M}(T_j(s), s)^+$ is the right derivative of $M(t,s)$ with respect to time at $t = T_j(s)$ and $c^+(T_j(s))$ is consumption just after the transfer.

The agents choose consumption $c(t,s)$, hours of work $h(t,s)$, money $M(t,s)$, and the transfer times $T_j(s)$, $j = 1, 2, \ldots$ They make this decision at time zero given the paths of the interest rate and of the price level. The maximization problem of agent $s = (M_0, k_0, B_0)$ is then given by

$$\max_{c,h,M,T_j} \sum_{j=0}^{\infty} \int_{T_j(s)}^{T_{j+1}(s)} e^{-\rho t} u(c(t,s), h(t,s))dt \quad (6)$$

subject to (4), (5), $M(t,s) \geq 0$, and $T_{j+1}(s) \geq T_j(s)$, given $M_0, k_0, B_0 \geq 0$. The parameter $\rho > 0$ is the intertemporal rate of discount. The utility function is $u(c(t,s), h(t,s)) =$

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$\log c(t, s) + \alpha_h \log (1 - h(t, s))$. Preferences are a function of consumption and hours of work only. In particular, the transfer cost does not enter the utility function as we want to emphasize the connection between the use of real resources and the management of money holdings.[10]

As bonds receive interest and money does not, the agents transfer the exact amount of money needed to consume until the next transfer. That is, the agents adjust $M^+(T_j)$, T_j, and T_{j+1} to obtain $M^-(T_{j+1}) = 0$, $j \geq 1$. We can still have $M^-(T_1) > 0$ as M_0 is given rather than being a choice. Using (5) and $M^-(T_{j+1}) = 0$ for $j \geq 1$, money just after the transfer at T_j is

$$M^+(T_j(s), s) = \int_{T_j}^{T_{j+1}} P(t)c(t, s)dt, \ j = 1, 2, \ldots \tag{7}$$

The government makes consumption expenditures G, makes transfer payments T, taxes labor income at the rate τ_L, and issues nominal bonds $B(t)$ and money $M(t)$. The transfer payments are made in lump sum form. The government controls the aggregate money supply at each time t by making exchanges of bonds and money in the asset markets. The financial responsibilities of the government at time t satisfy the period budget constraint

$$r(t)B(t) + P(t)G + P(t)T = \dot{B}(t) + \tau_L P(t)w(t)H(t) + \dot{M}(t). \tag{8}$$

That is, the government finances its responsibilities $r(t)B(t) + P(t)G + P(t)T$ by issuing new bonds, using revenues from labor taxes, and by issuing money. With the condition $\lim_{t\to\infty} B(t)e^{-rt} = 0$, the government budget constraint in present value is given by

$$B_0 + \int_0^\infty Q(t)P(t)(G + T)dt = \int_0^\infty Q(t)\tau_L w(t)H(t)dt + \int_0^\infty Q(t)P(t)\frac{\dot{M}(t)}{P(t)}dt. \tag{9}$$

The market clearing conditions for money and bonds are $M(t) = \int M(t, s)dF(s)$ and $B_0 = \int B_0(s) dF(s)$, where F is a given distribution of s. The market clearing condition for goods takes into account the goods used to pay the transfer cost. Let $A(t, \delta) \equiv \{s : T_j(s) \in [t, t + \delta]\}$ represent the set of agents that make a transfer during $[t, t + \delta]$. The number of goods used on average during $[t, t + \delta]$ to pay the transfer cost is then given by $\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s)$.

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[10]These preferences are derived from the King et al. (1988) preferences $u(c, h) = \frac{[c(1-h)^\alpha]^{1-1/\eta}}{1-1/\eta}$, with $\eta \to 1$, which are compatible with a balanced growth path. We also solved a version of the model with $\eta \neq 1$ and a version with Greenwood et al. (1988) preferences. Silva (2012) considers indivisible labor. We consider cash and credit goods in section 5. These changes do not affect our conclusions.

Taking the limit to obtain the number of goods used at time t yields the market clearing condition for goods, given by $\int c(t,s)dF(s) + \dot{K}(t) + \delta K(t) + G + \lim_{\delta \to 0} \int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s) = Y$. The market clearing conditions for capital and hours of work are $K(t) = \int k(t,s)dF(s)$ and $H(t) = \int h(t,s)dF(s)$.

An equilibrium is defined as prices $P(t)$, $Q(t)$, allocations $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, transfer times $T_j(s)$, $j = 1, 2, \ldots$, and a distribution of agents F such that (i) $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, and $T_j(s)$ solve the maximization problem (6) given $P(t)$, $r(t)$, and $r^k(t)$ for all $t \geq 0$ and s in the support of F; (ii) the government budget constraint holds; and (iii) the market clearing conditions for money, bonds, goods, capital, and hours of work hold.

3   Solving the Model

As we study the long run effects of policy toward taxes or inflation, we focus on an equilibrium in the steady state. In this equilibrium, the nominal interest rate is constant at r and the inflation rate is constant at π. The aggregate quantities of capital and labor are constant at K and H, and output is constant.

The transfer cost Γ and the interest payments on capital and bonds induce agents to follow (S,s) policies on consumption, money, capital claims, and bonds. For money, agent s makes a transfer at T_j to obtain money $M^+(T_j,s)$ at the beginning of a holding period. The agent then lets money holdings decrease until $M(t,s) = 0$, just before a new transfer at T_{j+1}. Symmetrically, individual bond holdings $B^+(T_j,s)$ are relatively low at T_j; they increase at the rate r until they reach $B^-(T_{j+1},s)$, just before T_{j+1}. The same applies to the behavior of $k(t,s)$. We assume that all agents behave in the same way in the steady state, in the sense that they follow the same pattern of consumption along holding periods. With constant inflation and interest rates, this implies that the agents start a holding period with a certain value of consumption, $c^+(T_j,s)$, which decreases until the value $c^-(T_{j+1},s)$, just before the transfer at T_{j+1}. The agents look the same along holding periods, although in general they are in different positions of the holding period.[11]

In the steady state, agents follow the same pattern of consumption along holding periods

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[11]For a description of different applications of (S,s) models in economics, see Caplin and Leahy (2010). See Alvarez et al. (2017) for the relation between inventory theoretical models of the demand for money and the welfare cost of inflation.

and choose the same interval between holding periods N. Let $n \in [0, N)$ denote the position of an agent along a holding period and reindex agents by n. Agent n makes the first transfer at $T_1(n) = n$, and then makes transfers at $n + N$, $n + 2N$ and so on. Given that the agents have the same consumption profile across holding periods, the distribution of agents along $[0, N)$ compatible with a steady state equilibrium is a uniform distribution, with density $1/N$. We can then solve backwards to find the initial values of M_0, B_0, and k_0 for each agent $n \in [0, N)$ that implies that the economy is in the steady state since $t = 0$.[12]

To characterize the pattern of consumption of each agent, consider the first order conditions of the individual maximization problem (6) with respect to consumption. These first order conditions imply

$$c(t, n) = \frac{e^{-\rho t}}{\lambda(n) P(t) Q(T_j)}, \quad t \in (T_j, T_{j+1}), \ j = 1, 2, \dots, \tag{10}$$

where $\lambda(n)$ is the Lagrange multiplier associated to the budget constraint (4). Let c_0 denote consumption just after a transfer. In the steady state, $P(t) = P_0 e^{\pi t}$, for a given initial price level P_0, and $Q(T_j) = e^{-rT_j}$. Therefore, rewriting (10), individual consumption along holding periods is given by

$$c(t, n) = c_0 e^{(r - \pi - \rho) t} e^{-r(t - T_j)}, \tag{11}$$

for $t \in [T_j(n), T_{j+1}(n)]$. We find aggregate consumption by integrating (11), using the fact that the distribution of agents along $[0, N)$ is uniform. Aggregate consumption is then

$$C(t) = c_0 e^{(r - \pi - \rho) t} \frac{1 - e^{-rN}}{rN}. \tag{12}$$

As aggregate consumption is constant in the steady state, the nominal interest rate and the inflation rate that are compatible with the steady state are such that $r = \rho + \pi$.

From (11) and $r = \rho + \pi$, we obtain that individual consumption $c(t, n)$ decreases during the holding period $t \in [T_j, T_{j+1})$ at the rate r. On the other hand, from (12), we obtain that aggregate consumption is constant at $c_0 \frac{1 - e^{-rN}}{rN}$. The individual behavior given by $c(t, n)$ is very different from the aggregate behavior, as in other (S, s) models. In particular, the

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[12]See Silva (2011) for additional results on the distribution of agents and for the characterization of M_0 and B_0 for each agent n. The characterization of k_0 is obtained analogously. We obtain government bonds B_0 by the government budget constraint.

variability of consumption is much larger at the individual level.

A similar situation happens with bonds. During holding periods, individual bond holdings follow equation (1), which implies $\dot{B}(t,n)/B(t,n) > r$. However, aggregating bond holdings across agents imply that aggregate bond holdings grow at the rate of inflation, $\dot{B}(t)/B(t) = \pi$. As $r = \rho + \pi$, individual bond holdings grow at a higher rate than aggregate bond holdings during holding periods. At the transfer dates T_j, bond holdings decrease sharply as the agent sells bonds for money and transfers money to the bank account. As $B(t)$ grows at the rate of inflation, the value of aggregate bond holdings is constant in real terms. Figure 1 shows the evolution of individual bond holding for two agents, n and n'. Agent n makes the first transfer at $t = 0$ and the second transfer at $T_2(n) = N$. Agent n' makes the first transfer at $T_1(n') > 0$ and the second transfer at $T_2(n') = T_1(n') + N$. The fact that $T_1(n) = 0$ implies that agent n starts with zero money holdings and so the agent needs to make a transfer at $t = 0$. Agent n' starts with some money, which delays the first transfer. At any time $t \geq 0$, the agents have different quantities of bonds, $B(t,n)$ and $B(t,n')$.
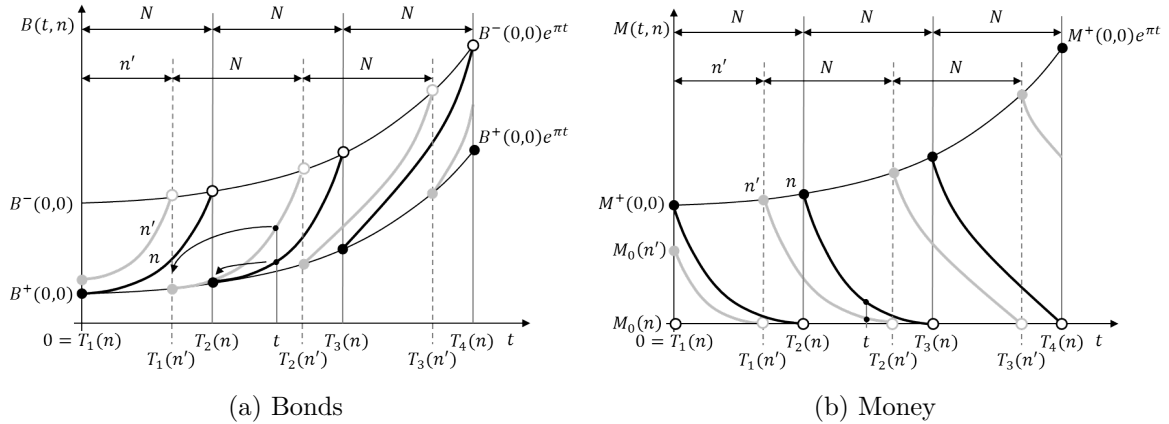

(a) Bonds　　　　　　　　　　　　　(b) Money

Figure 1: Individual bond and money holdings for two agents, n and n'. Agent n makes the first transfer at time zero; this agent starts with $M_0(n) = 0$. Agent n' starts with $M_0(n') > 0$ and makes the first transfer at $T_1(n') > 0$. At t, bond and money holdings are a function of the elapsed time since the last time the portfolio was rebalanced. Money at the beginning of holding periods increase at the rate of inflation, π. Individual bond holdings grow at the rate r and aggregate bond holdings grow at the rate π.

Given the production function $Y = Y_0 K^\theta H^{1-\theta}$, profit maximization implies prices $w = (1-\theta)Y_0(K/H)^\theta$ and $r^k = \theta Y_0(K/H)^{-(1-\theta)}$, which are constant in the steady state. With the non-arbitrage condition $r^k - \delta = r - \pi$ and $r = \rho + \pi$, we have $K/H = [\theta Y_0/(\rho + \delta)]^{1/(1-\theta)}$. Therefore, $K/Y = \theta/(\rho + \delta)$. As K is constant in the steady state, the investment output

ratio $(\dot{K} + \delta K)/Y$ is equal to $\delta\theta/(\rho + \delta)$.

The first order conditions for $h(t,n)$ imply

$$1 - h(t,n) = \frac{\alpha_h c_0}{(1 - \tau_L)w}, \qquad (13)$$

which is constant along holding periods, as wages are constant in the steady state. As there is a unit mass of agents, $H = h$. With the expression of wages, we obtain the equilibrium value of hours of work,

$$h = 1 - \frac{\alpha_h c_0}{(1 - \tau_L)(1 - \theta)Y_0(K/H)^\theta}. \qquad (14)$$

As c_0 depends on r and N, equation (14) determines hours of work as a function of r and N.

The market clearing condition for goods in the steady state is $C(t) + \delta K + G + \frac{1}{N}\Gamma = Y$. This equation implies

$$c_0\frac{1 - e^{-rN}}{rN} + \delta K + G + \frac{1}{N}\Gamma = Y. \qquad (15)$$

Dividing by Y, we obtain an expression for the consumption-income ratio $\hat{c}_0 \equiv c_0/Y$ in terms of N and the ratio between government spending and output,

$$\hat{c}_0\frac{1 - e^{-rN}}{rN} + \delta\frac{K}{Y} + \frac{G}{Y} + \frac{1}{N}\frac{\Gamma}{Y} = 1. \qquad (16)$$

The optimal holding period N is obtained with the first order conditions for $T_j(n)$. As derived in the appendix, N must satisfy

$$c_0(r, N)rN\left(1 - \frac{1 - e^{-\rho N}}{\rho N}\right) = \rho\Gamma. \qquad (17)$$

The aggregate demand for money is given by $M(t) = \frac{1}{N}\int M(t,n)\,dn$. Individual money holdings at t, $M(t,n)$, are obtained with the cash-in-advance constraint (5), given individual consumption $c(t,n)$ for an agent that has made a transfer at $T_j(n)$, $M(t,n) = \int_t^{T_{j+1}(n)} P(\tau)c(\tau,n)\,d\tau$, $\tau \in (T_j, T_{j+1})$. At any time t, there will be agents in their holding period $j+1$ and others in their holding period j. Taking this fact into account and the behavior of $c(t,n)$ in (11), it is possible to express real money holdings $\tilde{m} \equiv \frac{M}{P}$ in terms of r,

14

N, and c_0. As derived in the appendix,

$$\tilde{m}(r) = \frac{c_0(r,N)e^{-rN(r)}}{\rho} \left[\frac{e^{rN(r)} - 1}{rN(r)} - \frac{e^{(r-\rho)N(r)} - 1}{(r-\rho)N(r)} \right], \tag{18}$$

where $\tilde{m}(r)$ and $N(r)$ depend on the nominal interest rate r. Dividing (18) by Y yields the money-income ratio $m(r) \equiv M/(PY)$. For a constant value of m in the steady state, as output is constant in the steady state, the growth rate of money must be equal to the inflation rate π.[13]
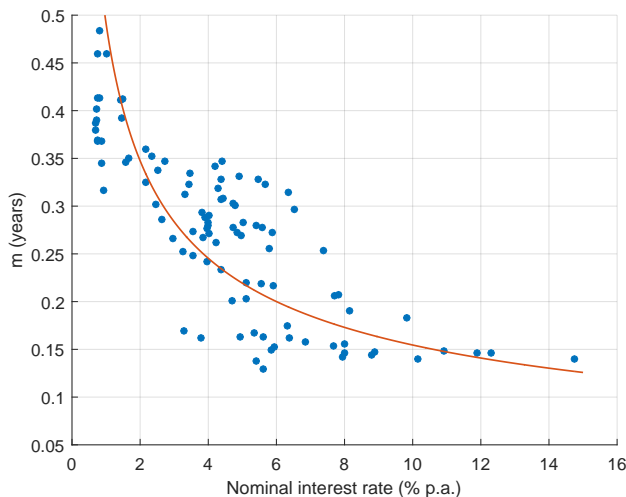

Figure 2: Money-income ratio implied by the model and U.S. annual data. M1 for the monetary aggregate and commercial paper rate for the nominal interest rate.

Figure 2 shows the money-income ratio $m(r)$ predicted by the model along with U.S. data.[14] The dataset is similar to the one used in Lucas (2000), Lagos and Wright (2005) and others.[15] The results for $m(r)$ imply an interest-elasticity of $-1/2$, which agrees with the

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[13]When Y grows at a constant rate, c_0 grows at the same rate of Y and the money-income ratio is constant.

[14]$m(r)$ is calibrated to pass through the geometric mean of the nominal interest rates and the money-income ratio in the data, as described in the next section. In the figure, spending is in the form of transfers and seigniorage is calculated with the nominal interest rate; this implies $\tau_L = 29.8$ percent. The values of τ_L and spending in the figure are then maintained constant for each value of r (the figure changes little if the increase in r is compensated with a decrease in τ_L to maintain constant government revenues). With fixed N, $m(r)$ has elasticity close to zero, which implies a straight line passing through the geometric mean of the data. It is possible to obtain elasticity of the demand for money with N fixed and cash goods only. However, this implies implausible values for parameters such as the elasticity of intertemporal substitution.

[15]The nominal interest rate is given by the commercial paper rate and the monetary aggregate is given by M1. Data are annual from 1900 to 1997 (the last year in which the commercial paper data are available from the same source). M1 and commercial paper rate were also used by Dotsey and Ireland (1996), Lucas (2000), Lagos and Wright (2005), Craig and Rocheteau (2008), among others. We use a similar dataset to facilitate the comparison of results. As it is known, money demand changed substantially at the beginning of 1990s.

findings in Guerron-Quintana (2009), Alvarez and Lippi (2009) and others.[16]

The equation that closes the model links government spending with tax revenues. The government budget constraint (8) implies

$$rb + G + T = b\frac{\dot{B}}{B} + \tau_L wH + \frac{M}{P}\frac{\dot{M}}{M}, \tag{19}$$

where $b = \frac{B(t)}{P(t)}$. As $\frac{\dot{B}}{B} = \pi$ and $\frac{\dot{M}}{M} = \pi$ in the steady state, we obtain that government consumption and transfers must satisfy the constraint

$$G + T + (r - \pi)b = \tau_L wH + \pi\frac{M}{P}. \tag{20}$$

That is, government spending plus interest payments must be financed through revenues from labor income taxes $\tau_L wH$ or through seigniorage $\pi\frac{M}{P}$. In this formulation it is assumed that real money is not a liability of the government, as the government does not need to finance interest payments on that liability.

Alternatively, as in Walsh (2010), add $(r - \pi)\frac{M}{P}$ to both sides of (20). It implies

$$G + T + (r - \pi)d = \tau_L wH + r\frac{M}{P}, \tag{21}$$

where $d = b + M/P$ is the total real public debt, which includes government real bonds and real money. This alternative way of writing the budget constraint conveys the message that if the government pays interest when it borrows with money then seigniorage would be $S = r\frac{M}{P}$, with the nominal interest rate as the analogous to a tax rate on real money holdings. Formulation (21) emphasizes that the total financing needs of the government include real government debt and real money, and that there are revenues from seigniorage even when inflation is equal to zero. This happens because, if the government finances itself with money, it does not pay interest on the quantity of money issued.

In the exercises below, we consider two cases: $b = 0$ and $d = 0$. When $b = 0$, the

Teles and Zhou (2005) and Lucas and Nicolini (2015) propose new monetary aggregates to account for the changes in the demand for money. Changing the monetary aggregate does not change our conclusion that the welfare cost of financing the government with inflation with N endogenous increases substantially.

[16] Our objective is to calculate the long-run consequences of taxes and inflation. Although idiosyncratic shocks might have an impact on the decision of the holding periods (Alvarez et al. 2011), we consider that this effect is smaller for our purposes than the long-run effects of interest rates.

government budget constraint becomes

$$G + T = \tau_L w H + \pi \frac{M}{P}. \tag{22}$$

In this formulation, seigniorage is defined as $S = \pi \frac{M}{P}$. This formulation is used in Sargent and Wallace (1981), Cooley and Hansen (1991, 1992), and others. In this case, the inflation rate is the analogous to a tax rate on real money holdings.

On the other hand, Chari et al. (1996), De Fiore and Teles (2003), and others assume that nominal assets $b + M/P$ are equal to zero, that is, $d = 0$. This implies

$$G + T = \tau_L w H + r \frac{M}{P}. \tag{23}$$

Equations (22) or (21) complete the characterization of the equilibrium. As the literature defines seigniorage as $S = \pi \frac{M}{P}$ or $S = r \frac{M}{P}$, we will make separate simulations for both definitions.[17] This characterization implies five equations (equations 14, 15, 17, 18, and either 22 or 23, depending on the definition of seigniorage) and five equilibrium variables (N, h, c_0, $\tilde{m}$, and τ_L or r, depending on the method of financing). Equilibrium output is obtained by $Y = Y_0 (K/H)^\theta h$, where Y_0 is normalized to 1. The initial price in equilibrium is obtained by setting an initial value for the money supply. We take as exogenous the values for government consumption G and government transfers T.

It is obtained below that financing through inflation a given increase in government expenditures is less costly for welfare when $b = 0$ than when $d = 0$. The intuition for this result can be grasped using equation (21) for the steady state, which implies $r - \pi = \rho$. A given increase in r affects the right-hand side of this equation in the same way for $b = 0$ or $d = 0$. On the other hand, as M/P decreases with an increase in r, the left-hand side decreases for the case with $b = 0$ (as $d = b + M/P$) and it does not change with $d = 0$. The left-hand side represents the financing needs of the economy. Therefore, for $b = 0$, an increase in r decreases the financing needs of the economy and the need of distortionary taxes.[18]

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[17]The definition of seigniorage with π or r are frequently found in the literature. The formulation with r is more common in the optimal taxation literature.

[18]Another way of understanding the result of smaller welfare cost with $b = 0$ is that the decrease in M/P in the left-hand side of equation (21) is equivalent to lump sum taxation of initial money holdings. The equilibrium with higher inflation would be obtained with lump sum taxes on initial money holdings followed by higher inflation. This effect decreases the welfare cost of inflation as as lump sum taxation decreases distortions. As

4 An Increase in Government Spending

We calculate the effects of an increase in government spending in the form of transfers T or in the form of government consumption expenditures on goods and services G. The economy is initially in a long run equilibrium, following the equilibrium equations described in section 3. After a change in the value of G or T, we recalculate the equilibrium values of τ_L and r so that the system of equations is satisfied for the new value of government spending.

We change τ_L and r separately. For financing the increase in spending with labor income taxes, we maintain the value of r at its initial value and find τ_L so that the government budget constraint and the remaining equilibrium equations are satisfied. Analogously, for financing the increase in spending with inflation, we maintain τ_L and find the interest rate r such that the equilibrium equations are satisfied. The new inflation rate is given by $r - \rho$.

4.1 Welfare Cost

Let the vector $\Psi_i = (G, T, \tau_L, r)$ denote a fiscal policy i. Let $c_i = c(\Psi_i)$ and $h_i = h(\Psi_i)$ denote the equilibrium profiles of consumption and hours of work for all agents in the support of F under policy $i = A$ or B. The welfare cost of a fiscal policy A with respect to a baseline fiscal policy B is defined as the income compensation w_A that leaves agents indifferent between an economy under fiscal policy A and economy under fiscal policy B. The value of w_A is such that $\tilde{U}[(1 + w_A)c_A, h_A] = \tilde{U}(c_B, h_B)$, where $\tilde{U}$ is the aggregate utility with equal weights for all agents.[19] The preferences $u(c, h) = \log c + \alpha_h \log(1 - h)$ imply

$$1 + w_A = \frac{c_0(\Psi_B)}{c_0(\Psi_A)} \left(\frac{1 - h(\Psi_B)}{1 - h(\Psi_A)} \right)^{\alpha_h} \exp \left(\frac{r(\Psi_A)N(\Psi_A)}{2} - \frac{r(\Psi_B)N(\Psi_B)}{2} \right). \qquad (24)$$

The values of $c_0(\Psi_i)$, $h(\Psi_i)$, $N(\Psi_i)$, and $r(\Psi_i)$, $i = A, B$, are given by the equilibrium conditions in section 3. They depend, in particular, on the size of the government spending and on the way in which the government is financed.

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stated in Chari et al. (1996) and in others, it is optimal to tax away through inflation any initial positive value of bonds and money. Therefore, with $b = 0$, the government could tax lump sum initial monetary holdings. With $d = 0$, the government cannot tax lump sum existing bonds or money. It is common in the literature to set $d = 0$ to emphasize that the government can only resort to distortionary taxation.

[19]$\tilde{U}(\tilde{c}, \tilde{h}) = \int \int_0^\infty e^{-\rho t} u(c(t, n), h(t, n)) dt dF(n)$, where $\tilde{c}$ and $\tilde{h}$ denote consumption and hours of work for all agents in the support of F and $t \in [0, \infty)$. The definition of w_A uses the fact that consumption is homogeneous of degree one in income.

Government consumption G does not enter the utility function. Therefore, an increase in G always implies a positive welfare cost with respect to the economy with lower G. Government consumption enters the market clearing condition and decreases the availability of private consumption for the same output. However, we can compare an economy with the same value of government consumption, but in which A denotes financing with inflation and B denotes financing with labor income taxes. If w_A is positive for this case, then the interpretation is that agents would be better off if the government financed government consumption with taxes rather than with inflation.

We also consider an increase in transfers T. Transfers are distributed to agents in lump sum form through the individual budget constraints. On the other hand, the government needs to increase distortionary taxes or inflation to finance the additional transfers. The government taxes the economy in a distortionary way and redistributes the tax revenues in lump sum form.

As for government consumption, we can calculate the welfare cost of changing the method of financing given the same level of transfers. For a given level of transfers T, let economy A finance the transfers with inflation and economy B with taxes. If $w_A > 0$ then agents would be better off by substituting inflation with taxes.

The two forms of increasing government spending, through government consumption or transfers, yield different results and allow the analysis of different aspects of fiscal policy. When the government increases G, for example, we can study the government consumption multiplier. Also, we can analyze the behavior of the multiplier when the increase in government consumption is financed with inflation or with taxes.

Adão and Silva (2019) show that in a cash-in-advance economy it is optimal to finance transfers with inflation. This case has the counterpart here of an economy with fixed holding periods. As a result, we will see that the welfare cost of substituting inflation with taxes is negative for an economy with fixed holding periods. In contrast, the welfare cost is positive when the holding periods are endogenous.

4.2 Parameters

We set $\theta = 0.36$ for the parameter for capital in the production function and $\delta = 0.10$ for depreciation (Cooley and Hansen 1989). We set $\rho = 3$ percent per year (Lucas 2000). The parameters α_h and Γ are set so that hours of work are equal to 0.3 and the money-income ratio $m(r)$ passes through the geometric mean of the data; that is, $r_{avg} = 3.64$ percent p.a. and $m_{avg} = 0.257$ year. The data on money-income ratio imply that the average person in the U.S. holds about one quarter of income in money, or that average velocity is about $1/0.25 = 4$ per year).[20] Figure 2 shows the money-income implied by the model together with the data.

We set the initial value of government spending such that the initial ratio of government spending to output is equal to 20 percent.[21] The value of τ_L is obtained so that equations (22) or (23) for the government budget constraint are satisfied. We set the initial value for seigniorage using the mean of the nominal interest rate. This procedure yields an initial value for τ_L equal to 29.79 percent when seigniorage is given by $S = rM/P$ and equal to 30.99 percent when $S = \pi M/P$. Initial seigniorage is equal to 0.94 percent when $S = rM/P$ and to 0.16 percent when $S = \pi M/P$.

We depart from other calibrations in the determination of τ_L as we obtain τ_L such that it satisfies the government budget constraint. We do not take this value from estimates of marginal tax rates. In particular, there are no lump sum taxes to close the government budget constraint. Nevertheless, the value of τ_L is close to the ones in other papers. For example, $\tau_L = 23$ percent in Cooley and Hansen (1992). Carey and Rabesona (2002) estimate an effective labor tax rate of 23.4 percent for the U.S. and a combined consumption and labor tax rate of 28.3 percent.

As hours of work and the money-income ratio are equilibrium variables, the values of α_h and Γ so that $h = 0.3$ and $m = m_{avg}$ vary if government spending is in the form of transfers or government consumption; and if seigniorage is defined with the inflation rate or with

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[20]Analogously, Alvarez et al. (2009) parameterize the holding period (exogenous in their case) such that the theoretical demand for money approximates the average velocity in the data. Also, Lucas (2000) determines the parameters for the demand for money such that the theoretical demand for money passes through the geometric average of the data.

[21]The average government spending to output ratio for the United States from 1950 to 2016 is 20.9 percent, where government spending is given by government consumption and investment (data from the St. Louis Fed). We also obtained the predictions for different values of the government-output ratio such as zero, 5 percent, and 10 percent. The qualitative results do not change.

the nominal interest rate. The values, however, do not vary much. The value of α_h varies from 1.45 for the case of an increase in transfers and $S = rM/P$, the case discussed in the introduction, to 2.00, for the case of an increase in government spending and $S = rM/P$. The value of Γ in these two cases is 36.65 and 51.16 respectively. To have an idea of the meaning of these values, consider the ratio Γ/Y, which yields the cost of a transfer in working days. This ratio is equal to 2.49 with transfers and 3.41 with government consumption.[22] These values imply infrequent transfers from the brokerage account to the bank account, as the initial value of N is equal to 367 days with government consumption and 264 days with transfers. When inflation increases to finance the increase in government spending, the values of N, when endogenous, decrease to 197 days and 127 days.[23]

We can also evaluate the transfer cost by calculating the minutes per week devoted to financial services. As Γ/Y is the cost of transfers in working days and $1/N$ is the number of transfers per year, then $\Gamma/Y \times 1/N$ yields the cost of transfers in working days per year.[24] This measure implies that about 0.95 percent of working days is devoted to financial services in the steady state.[25] To obtain the time devoted to financial transfers implied by the model, we multiply this value by hours of work. The average weekly hours of work in the U.S. from 1957 to 1997 is 36.5 hours per week (data from the OECD). The model then implies 21 minutes per week for the time devoted to financial services.

4.3 An Increase in Government Spending

We now set the economy with the initial value for the ratio of government spending to output equal to 20 percent and increase the value of government spending by 5 percent. This change increases the ratio of government spending to output by about 1 percentage point, from 20

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[22]We also calculated the results using $\Gamma = \gamma Y$ for the cost parameter, which implies a demand for money homogeneous of degree 1 in income. However, as Y is an equilibrium value, γY varies with the policy. In spite of these differences, the values of γ and of Γ/Y are approximately equal. The simulation results are similar with Γ or $\Gamma = \gamma Y$.

[23]For comparison, Alvarez et al. (2009) set N equal to 24 months and 36 months (higher values because Alvarez et al. use M2 instead of M1 and allow for cash inflows). As in other models with market segmentation, the value of N implied by the parameters is large. There is evidence that firms and households in fact rebalance their portfolios infrequently (Christiano et al. 1996, Vissing-Jorgensen 2002, Alvarez et al. 2009). Agents maintain large money holdings even when they pay high interest rates for credit cards (Telyukova 2013).

[24]Γ is in goods per transfer and Y is in goods per working day. So, Γ/Y is in working days per transfer.

[25]Using the equilibrium values of Y and N, we obtain values between 0.94 and 0.95 for the four cases considered in the simulations (the four combinations between government consumption and transfers and seigniorage defined with inflation or nominal interest rates).

percent to 21 percent.[26]

We study two ways of financing the increase in spending: by increasing labor income taxes τ_L and by increasing the inflation rate π. We consider seigniorage to be given by $S = rM/P$ or $S = \pi M/P$. The increase in government spending can be in the form of transfers or in the form of government consumption. The initial equilibrium is for the economy under the benchmark parameterization with a balanced government budget constraint.

The results are in tables 1–3 and in figures 3–6. We study the effects of the change in policy using the model with a fixed N and with endogenous N. A varying financial sector is represented by an endogenous N. The value of N when it is fixed is equal to the optimal value of N for the initial situation. All other parameters are the same for the cases with N fixed and N endogenous. The only difference between the two cases is that N is allowed to change optimally in the case of N endogenous. The definition of seigniorage has a small impact. The form of the increase in government spending, either transfers or government consumption, the method of financing, and having N fixed or endogenous are more important for the results.[27]

Table 1: Welfare cost of an increase in government spending

Values in % of income	Model and Method of Financing					
	N Endogenous			N Fixed		
	Inflation	Labor Tax	From Tax to Inflation	Inflation	Labor Tax	From Tax to Inflation
Transfers, Seigniorage $r \times M/P$	1.45	0.95	0.50	0.49	0.96	-0.46
Government Cons, Seigniorage $r \times M/P$	3.03	2.11	0.90	2.01	2.13	-0.11
Transfers, Seigniorage $\pi \times M/P$	0.97	1.01	-0.04	0.51	1.01	-0.50
Government Cons, Seigniorage $\pi \times M/P$	2.41	2.13	0.28	2.00	2.13	-0.12

Government spending increases 5%. The government-output ratio increases from 20% to about 21%. Increase in government spending either in the form of transfers or government consumption. The calculation of the welfare cost follows equation (24). Segniorage is defined with either the nominal interest rate or with the inflation rate. From tax to inflation: welfare cost of changing the method of financing from labor income tax to inflation. A negative sign implies a welfare gain.

The case of fixed N approximates the standard cash-in-advance model with fixed periods.[28]

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[26]The ratio G/Y does not increase exactly to 21 percent because Y is endogenous. Y does not change much so that the new ratio increases to about 21 percent.

[27]Figure 6 shows the effects when seigniorage is defined with the interest rate. The qualitative results when seigniorage is defined with inflation are similar.

[28]It is not equal to a standard cash-in-advance model because the agents can still smooth consumption

In particular, the demand for money is approximately constant with N fixed. As it is known, an inflation tax or a labor income tax have similar effects in a cash-in-advance economy. In accordance with this result, the effects across simulations are similar with N fixed. What strongly changes predictions is the case in which N is endogenous. In this case, the demand for money decreases more strongly when inflation increases, a pattern compatible with the data. For N endogenous, financing the government with inflation or taxes have substantially different implications.

As shown in table 1, considering fixed periods underestimates the welfare cost of financing the increase in government spending with inflation. Consider the case of an increase in government spending in the form of transfers and seigniorage defined with the nominal interest rate, in the first row of table 1. Financing the increase in government spending with inflation implies a welfare cost of 1.45 percent using the model with endogenous periods. The welfare cost is only 0.49 percent with fixed intervals. The underestimation of the effects is such that a model with fixed N implies that there is a gain in financing the increase in government spending with inflation, shown by the negative value -0.46 in the table. The result is reversed with endogenous N. Considering endogenous holding periods in general increases the welfare cost by about one percentage point.

Figure 3 shows the effects on welfare in a different way.[29] The two top figures show the welfare cost associated with an increase in government spending financed with inflation for N endogenous and fixed and the welfare cost when the increase in spending is financed with taxes for N fixed (the results with N endogenous or fixed overlap when financing with taxes). The welfare cost with N endogenous is always higher than with N fixed. Moreover, the welfare cost of financing with taxes is higher than with inflation for N fixed. As a result, changing the fiscal policy from financing with taxes to inflation implies gains in welfare in a model with fixed N such as a cash-in-advance model.

The reason for the welfare gains of inflation predicted with N fixed is the existence of transfers or government consumption and the exclusion of the additional costs generated by the increase in the financial sector. Adão and Silva (2019) show that taking government

during the interval N. However, the effects in this model and in a model in which consumption is constant during holding periods are similar.

[29]The figure concentrates on the case in which seigniorage is defined as $S = rM/P$.
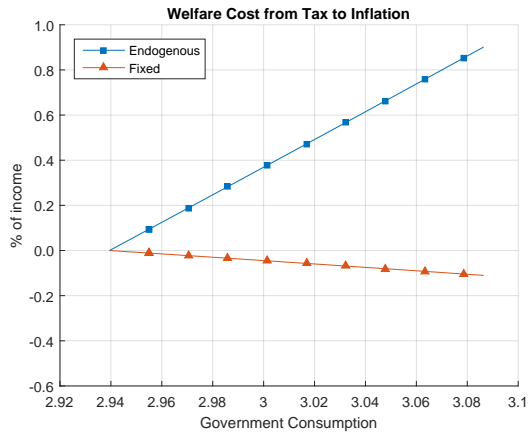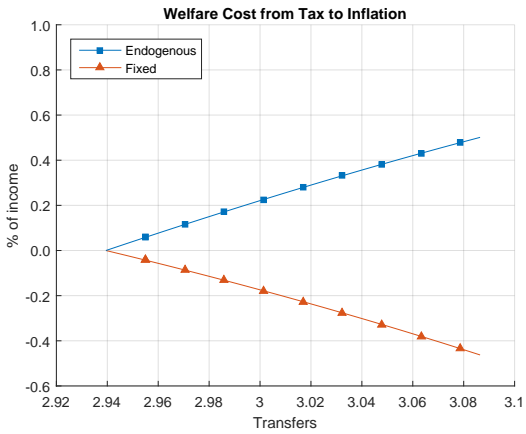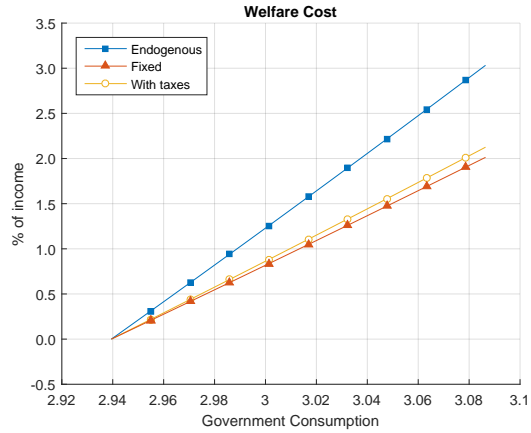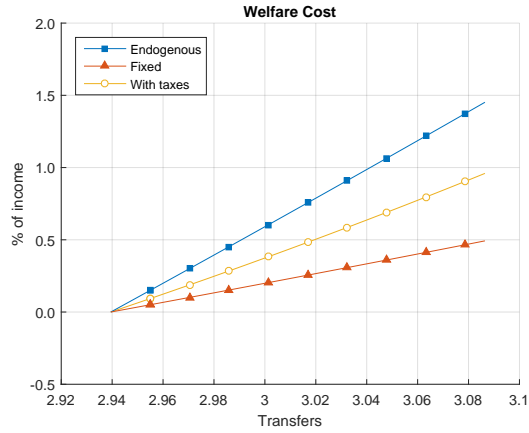
(a) Increase in transfers

(b) Increase in government consumption

Figure 3: Welfare cost of changes in policy. Endogenous or fixed N: financing with inflation. With taxes: N fixed, financing with taxes (endogenous or fixed N overlap in this case). Negative values imply a welfare gain of changing policy from taxes to inflation. Seigniorage defined as $S = rM/P$. Horizontal axes in units of the model. The initial values of transfers or government consumption correspond to 20 percent of GDP. Fixed holding periods implies gains of financing an increase in spending with inflation. With endogenous holding periods, there are substantial costs of financing with inflation.

transfers into account changes the optimal set of taxes for a cash-in-advance economy. As the government does not tax transfers directly, it is optimal to increase inflation to tax this additional income. As a result, the model with fixed N shows a gain of substituting labor taxes with inflation.

The model with N endogenous takes into account the costs of the increase in the use of financial services. These costs are greater than the gains of taxing the income from transfers. Agents use more intensively financial services with inflation to decrease their demand for money. When these costs are taken into account, the model with N endogenous predicts that it is optimal to finance the government with taxes.[30]

Table 2: Inflation and labor tax to finance an increase in government spending

	Model and Method of Financing			
	N Endogenous		N Fixed	
	Inflation, π (% p.a.)	Labor Tax, τ_L (%)	Inflation, π (% p.a.)	Labor Tax, τ_L (%)
Transfers, Seigniorage $r \times M/P$	12.72	32.12	5.47	32.14
Government Cons, Seigniorage $r \times M/P$	9.68	31.44	4.64	31.46
Transfers, Seigniorage $\pi \times M/P$	7.82	33.37	5.49	33.37
Government Cons, Seigniorage $\pi \times M/P$	5.79	32.65	4.59	32.64

Initial inflation rate: 0.64% p.a. (percent per annum). Initial labor tax: 29.79% (case rM/P) and 30.99% (case $\pi M/P$). See table 1 for welfare cost calculations and definitions.

Phelps (1973) states that positive inflation can be optimal when only distortionary taxes are available. More recent results point out that the Friedman rule is optimal in cash-in-advance models with distortionary taxation.[31] This is the case of Kimbrough (1986), Correia and Teles (1996, 1999), Chari et al. (1996), De Fiore and Teles (2003) and others. Adão and Silva (2019) show that the Friedman rule is not optimal in the context of Chari et al. (1996), even when both labor and consumption taxes are available, when there are positive government transfers.[32]

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[30]The gains of inflation with fixed N are smaller with government consumption. When we remove capital from the model, the model with fixed N and government consumption does not predict welfare gains of inflation. However, the difference between predictions with fixed N and and endogenous N is robust for the cases with or without capital.

[31]The Friedman rule (Friedman 1969), states the optimality of having nominal interest rates equal to zero.

[32]Da Costa and Werning (2008) have additional results on the Friedman rule. See Kocherlakota (2005) for a
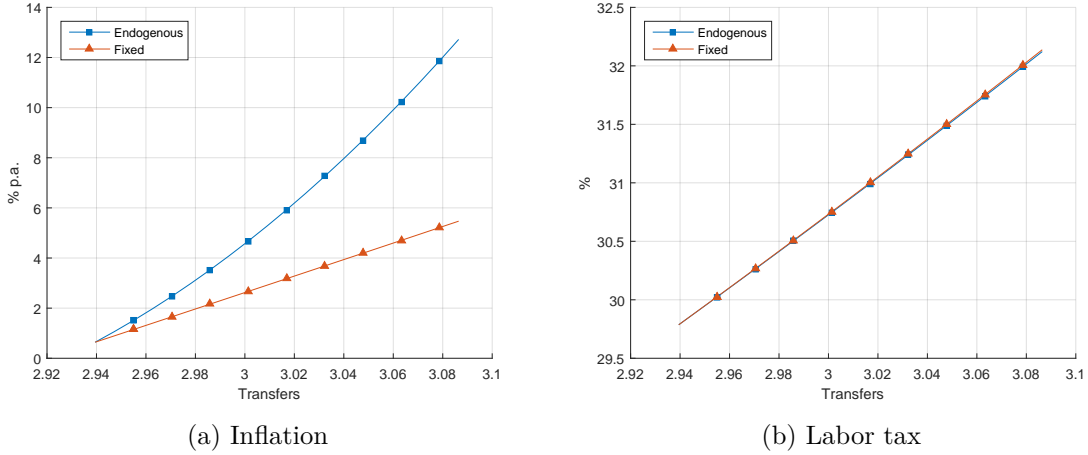
(a) Inflation   (b) Labor tax

Figure 4: Changes in inflation π and in the labor tax τ_L to finance an increase in transfers. The model with fixed holding periods predicts a smaller increase in inflation. Financing with taxes does not imply a change in inflation. As a result, the effects on the labor tax are similar for the two models. Seigniorage defined as $S = rM/P$. The qualitative results of an increase in government consumption and with $S = \pi M/P$ are similar.

Table 2 and figure 4 show that considering fixed periods underestimates the required inflation to finance the increase in government spending. For an increase in transfers, the required inflation is 12.72 percent per year with endogenous periods while it is 5.47 percent with fixed periods. With fixed periods, the demand for money does not change much with inflation. This implies that the government is able to finance itself with a smaller rate of inflation.

Table 3 shows revenues from seigniorage and from labor taxes. Government spending is still financed mostly with labor income taxes even when the government uses inflation to finance the increase in spending.[33]

When the policy change involves only an increase in taxes, the two models yield similar predictions. In figure 4, the cases with endogenous or fixed periods yield the same labor tax to finance the increase in spending. This is expected, as an increase as taxes have a small influence on the demand for money.

The predictions with N fixed or endogenous are different when the increase in spending

survey. Guerron-Quintana (2011) studies the welfare cost of inflation in a new Keynesian model. Wen (2015) studies the welfare cost of inflation when liquidity reduces consumption uncertainty. Our focus is on the effects of an increase in government spending financed in different ways.

[33]A different question is the transition toward the new policy. Studying the equilibrium transition of an aggregated (S, s) model is beyond the objective of this paper. Adão and Silva (2017) study the transition for a simplified version of the model. See also Gertler and Leahy (2008) and Stokey (2009).

Table 3: Revenues from seigniorage and labor taxes after an increase in government spending

Values in % of output	Model and Method of Financing							
	N Endogenous				N Fixed			
	Inflation		Labor Tax		Inflation		Labor Tax	
	$\tau_L wH$	Seig	$\tau_L wH$	Seig	$\tau_L wH$	Seig	$\tau_L wH$	Seig
Transfers, Seigniorage $r \times M/P$	19.06	1.93	20.56	0.95	19.06	2.19	20.57	0.94
Government Cons, Seigniorage $r \times M/P$	19.06	1.71	20.12	0.93	19.06	1.94	20.13	0.92
Transfers, Seigniorage $\pi \times M/P$	19.84	1.16	21.36	0.17	19.84	1.42	21.35	0.16
Government Cons, Seigniorage $\pi \times M/P$	19.84	0.94	20.89	0.16	19.84	1.16	20.89	0.16

G: government spending in the form of transfers or government consumption. The initial value of G is such that G/Y is equal to 20%. Initial revenues from labor taxes: 19.06% (rM/P) and 19.85% ($\pi M/P$). Initial revenues from seigniorage: 0.94% (rM/P) and 0.16% ($\pi M/P$).

is financed with inflation. In this case, the opportunity cost of holding money increases and, therefore, the agents spend resources to decrease their real demand for money. With fixed periods, the agents can still affect the demand for money by making consumption within holding periods steeper. This behavior also shows up with N endogenous, but most of the decrease in the demand for money is obtained by decreasing N. With N fixed, the money-income ratio decreases about 2 percent. With N endogenous, the money-income ratio decreases about 50%. An inflation of 12 percent p.a. implies a decrease in the money-income ratio to the values of the 1980s, corresponding to the data points on the Southeast of figure 2.

We interpret the increase in the trading frequency as an increase in the use of financial services. Let $\Gamma \frac{1}{N}$ be the measure of the financial system in the economy, as this is the cost paid by agents to transform bonds into money for each point in time. Using this measure, the value of financial services as a fraction of output increases from 0.95 percent to 1.97 percent for the case of an increase in transfers financed with inflation and seigniorage defined with the nominal interest rate. With seigniorage defined with inflation, the fraction of financial services to output increases from 0.16 percent to 1.16 percent. For the four cases considered, the fraction of financial services to output increases by about 1 percentage point. English (1999) provides evidence that the share of financial services in the economy increases with inflation. According to English, a 10 percent increase in inflation in U.S. would imply an

increase in the financial sector of 1.3 percentage points, which agrees with our results.[34]

(a) Increase in transfers   (b) Increase in government consumption
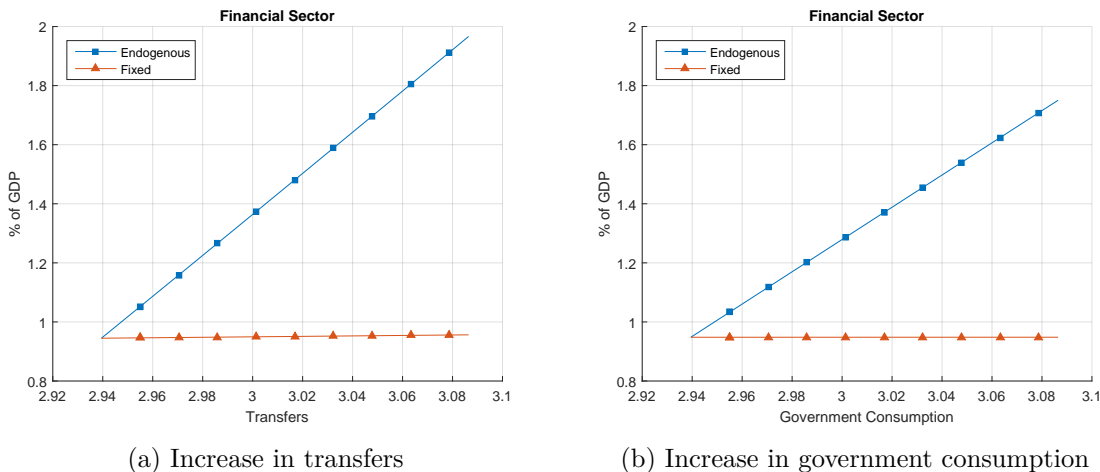
Figure 5: Size of the financial sector when the government finances an increase in spending with inflation, $(\Gamma \times 1/N)/Y$. Inflation increases about 10 percentage points in each case (table 2). With fixed periods, the model does not predict an increase in the financial sector. With endogenous periods, the financial sector increases about 1 percentage point, which agrees with the empirical evidence in English (1999) of an increase in inflation of 10 percentage points.

Figure 5 shows the size of the financial sector for each value of spending when the increase in government spending is financed with inflation. In accordance with the empirical evidence, the model with endogenous holding periods implies an increase in the financial sector as a percentage of GDP. With fixed holding period, the size of the financial sector, counterfactually, does not change. As stated above, the total increase in the financial sector implied by the model, of about 1 percentage point, corresponds to the empirical evidence on the increase in the financial sector when inflation increases 10 percentage points.[35]

An unexpected implication of considering N endogenous is the response of output after the increase in G. The response of output implies a government spending multiplier slightly above 1 for N endogenous when an increase in government consumption is financed with inflation. When the increase in G is financed with taxes, agents substitute away from labor toward leisure, which decreases output (columns 3 and 4 in figure 6). When N is fixed, inflation and taxes have similar effects.[36] When N is endogenous, the increase in inflation

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[34]The evidence that the share of financial services in the economy increases with inflation is also consistent with Aiyagari et al. (1998).

[35]The financial sector provides more services than it is implied by the model. In particular, the model abstract from risk sharing, capital allocation, and other financial services. We are concerned here with the increase in the financial sector solely given by the increase in inflation.

[36]This is the standard result in cash-in-advance models, in which inflation has effects similar to a labor tax.
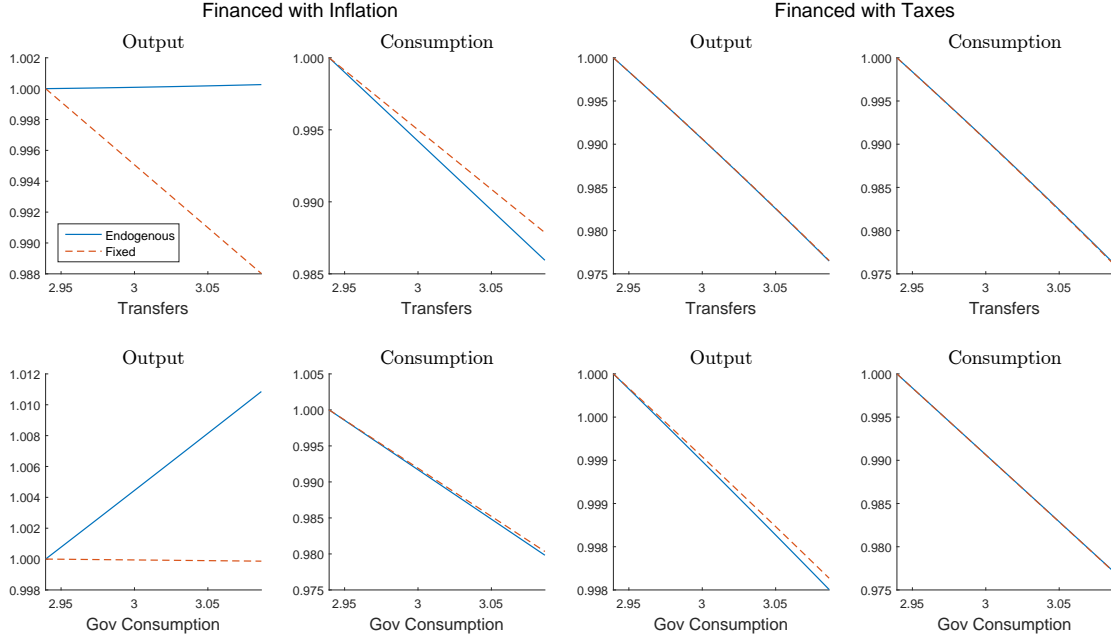
Figure 6: Effects on output and consumption relative to initial levels of an increase in spending financed in different ways. Top: increase in spending in the form of transfers. Bottom: increase in spending in the form of government consumption. Seigniorage $S = rM/P$.

makes agents decrease the demand for money, which requires an increase in the provision of financial services. The increase in financial services implies an increase in output. This effect implies a multiplier above 1 when the increase in government spending is made through consumption and financed with inflation (output increases in columns 1 and 2 of figure 6). When the increase in spending is in the form of transfers, the multiplier is smaller, but it is still positive. With transfers, the multiplier is equal to -1.2 with N fixed and 0.35 with N endogenous. With government consumption, the multiplier is equal to -0.01 with N fixed and 1.09 with N endogenous.[37]

A multiplier slightly above 1 is compatible with the values in Hall (2009) and Ramey (2011). Woodford (2011) discusses how the multiplier can be above 1 in models with sticky prices or sticky wages. Here, the multiplier is above 1 with flexible prices. The only friction in the model are financial frictions. Although output increases, the welfare cost of inflation is large. The overall effect is such that welfare decreases.

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[37]The multiplier is calculated by the ratio $\Delta Y/\Delta X$, where ΔY is the change in output obtained in the simulations and ΔX is the change in transfer payments or government consumption. We make Δ small and calculate the multiplier from the initial value of transfers payments or government consumption.

Having endogenous periods is especially relevant to study policy changes that imply changes in inflation. The effects on the welfare cost, output and consumption are approximately equal with N fixed or endogenous when the increase in government spending is financed with taxes. The predicted increase in τ_L is also similar in the two cases. The models with N fixed or endogenous yield similar predictions as inflation does not change when taxes are used to finance the increase in government spending.

Figure 7 shows the welfare cost of an increase in the nominal interest rate. The figure focuses on the case of seigniorage $S = r \times M/P$ and spending in the form of transfers. The model is calibrated as for the construction of figure 3. The difference is that now the level of transfers is maintained constant at its initial value and the nominal interest rate increases. Inflation also increases, as $\pi = r - \rho$. The additional revenues generated with seigniorage are used to decrease the labor income tax. The labor income tax decreases so that the government budget constraint is balanced.
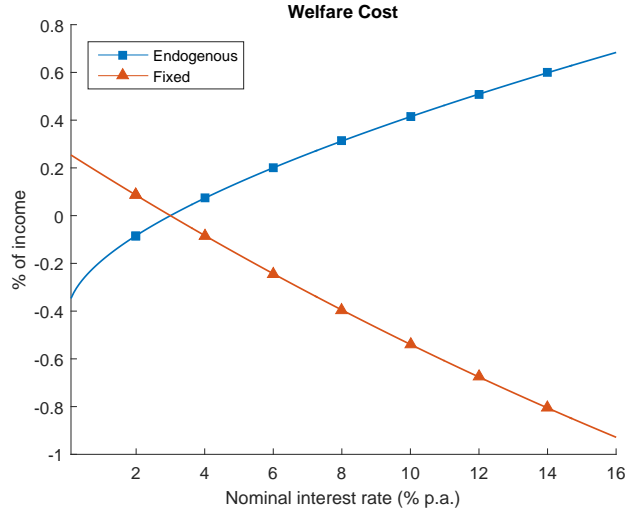

Figure 7: Welfare cost of an increase in the nominal interest rate r compensated by a decrease in the labor income tax τ_L so that the government budget constraint holds. Model with transfers and $S = r \times M/P$. Transfers set at 20 percent of output, which correspond to the initial value of transfers in figure 3, and maintained constant. The welfare cost is calculated with respect to $r = 3\%$ p.a. (zero inflation). As transfers are positive, the Friedman rule does not hold with fixed holding periods (positive welfare cost for $r = 0$). Moreover, it is optimal to increase inflation according to the model with fixed N (the welfare cost decreases with an increase in r). The model with endogenous N predicts a welfare loss of increasing inflation. At $r = 13$, the difference in predictions of the two models is $0.6 - (-0.7) = 1.3\%$ of income.

When the effect of the financial sector is not considered (N fixed), decreasing labor income

taxes and increasing higher inflation generates welfare gains. The gains are substantial. With $r = 13\%$ p.a. (ten percent inflation), the welfare gain is 0.7% of income. These gains are a result of the optimal indirect taxation of transfers through inflation[38] When the additional costs with the financial sector are taken into account (N endogenous), the welfare cost is always positive. With $r = 13\%$ p.a., the welfare cost is 0.6% of income. The difference in the predictions is 1.3% of income.

Endogenous periods matters crucially when the policy change involves an increase in inflation. In this case, the welfare cost of financing with inflation is larger with N endogenous. The inflation rate, output and consumption are also different with N fixed or endogenous. A model with N endogenous implies a better match on the behavior of the demand for money and on the increase in financial services after an increase in inflation. Given the results obtained here, it is important to consider changes in the frequency of trades to evaluate the effects of inflation. Especially, for the estimates of the welfare cost of different policy changes.

5 Cash and Credit Goods

We show above that the predicted welfare cost of financing the government with inflation is substantial using a model in which the financial sector increases with inflation. Changes in the financial sector are obtained with endogenous holding periods N. With fixed holding periods, the elasticity of the demand for money is small and the welfare cost of inflation is small compared with the welfare cost of increasing distortionary taxation. With fixed holding periods, it is optimal to increase inflation.

Introducing the decision between cash and credit goods increases the elasticity of the demand for money in standard cash-in-advance models. It could be expected that this extension would increase the welfare cost of financing through inflation with fixed N and make the models with fixed and endogenous N equivalent. This is not so. Cooley and Hansen (1991, 1992) have cash and credit goods. The elasticity of the demand for money is higher with credit goods, but the increase in elasticity is small. They still find that inflation is less costly in terms of welfare than labor taxes. In table 1 of Cooley and Hansen (1992), replacing

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[38]These welfare gains are compatible with the results of Adão and Silva (2019) and are similar in magnitude to the values found in Cooley and Hansen (1992).

labor income taxes with inflation generates a change in the welfare cost from 13.3 to 12.43 percent of income, which implies that decreasing labor taxes and increasing inflation generates a welfare gain of 0.87 percent of income with a standard cash-in-advance model with cash and credit goods.

Nevertheless, introducing cash and credit good in the context of our model could make the models with endogenous and fixed N similar. We verify this possibility. Introducing credit goods increases the elasticity of the demand for money, as expected, but the effect is small to change the conclusions of a model with fixed N, which does not take into account the increase in the financial sector. The model with fixed N implies a welfare cost of inflation smaller than the welfare cost of distortionary taxation. A model with endogenous N with cash and credit goods still implies that inflation is more costly than distortionary taxation.

The main change is that agents can now use the resources of the brokerage to purchase credit goods. Let preferences in (6) be now given by $\alpha_1 \log c_1 + (1 - \alpha_1) \log c_2 + \alpha_h \log(1 - h)$, where c_1 and c_2 respectively denote cash and credit goods and α_1 is the weight on cash goods. The evolution of bond holdings of (1) is now

$$\dot{B}(t,s) = r(t)B(t,s) + P(t)(1 - \tau_L)w(t)h(t,s) + T - P(t)c_2(t), \tag{25}$$

where spending on credit goods c_2 decreases bond holdings; as above, T denotes government transfers payments. Equation (4) assumes the form as above, where now $W_0(s) = B_0(s) + P_0 k_0(s) - \int_0^\infty Q(t)P(t)c_2(t,s)dt + \int_0^\infty Q(t)P(t)(1 - \tau_L)w(t)h(t,s)dt$.

The problem can be solved following the same steps as in section 3. In the steady state, it is convenient to index agents by their position in the holding period, $n \in [0, N)$. The first order conditions for cash and credit goods in the steady state imply

$$c_1(t,n) = \alpha_1 \frac{1}{P_0\lambda} e^{-r(t-T_j)}, \quad t \in [T_j, T_{j+1}), \ j = 1, 2, \dots \tag{26}$$

$$c_2(t,n) = (1 - \alpha_1)\frac{1}{P_0\lambda}, \tag{27}$$

where λ is the Lagrange multiplied associated with the budget constraint (4), with the value of W_0 with credit goods.[39]

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[39]As in section 3, substitute $P(t) = P_0 e^{\pi t}$ and $Q(t) = e^{-rt}$. In the steady state, $r = \rho + \pi$ and λ is equal

Consumption of cash goods decreases within holding periods, as in the case with cash goods only ($\alpha_1 = 1$). Agents economize on cash by concentrating spending on cash goods at the beginning of holding periods. On the other hand, consumption of credit goods is constant. Agents can maintain constant spending on credit goods because they can spend directly from interest-bearing bonds on credit goods.

Let c_{01} be the level of consumption on cash goods at the beginning of holding periods. According to (26), $c_{01} = \alpha_1/(\lambda P_0)$. The ratio between credit goods and the beginning level of cash goods is given by

$$\frac{c_2(t,n)}{c_{01}} = \frac{1 - \alpha_1}{\alpha_1}. \tag{28}$$

A decrease in α_1 decreases consumption of cash goods. As cash is only necessary to purchase cash goods, the demand for money also decreases with a decrease in α_1.

The system of equations that characterizes the equilibrium with cash and credit goods is given by

$$c_{01}(r,N)rN\left(1 - \frac{1 - e^{-\rho N}}{\rho N}\right) = \rho\Gamma \tag{29}$$

$$h = 1 - \frac{\alpha_h c_{01}(r,N)}{\alpha_1\left(1 - \tau_L\right)\left(1 - \theta\right)Y_0(K/H)^\theta} \tag{30}$$

$$c_{01}(r,N)\frac{1 - e^{-rN}}{rN} + \frac{1 - \alpha_1}{\alpha_1}c_{01}(r,N) + \delta K + G + \frac{1}{N}\Gamma = Y \tag{31}$$

$$c_2(r,N) = \frac{1 - \alpha_1}{\alpha_1}c_{01}(r,N) \tag{32}$$

$$G + T = \tau_L wH + r\tilde{m}(r) \tag{33}$$

$$\tilde{m}(r) = \frac{c_{01}(r,N)e^{-rN(r)}}{\rho}\left[\frac{e^{rN(r)} - 1}{rN(r)} - \frac{e^{(r-\rho)N(r)} - 1}{(r-\rho)N(r)}\right] \tag{34}$$

The system is similar to equations (14), (15), (17), (18), and (23). We have the extra equilibrium variable, c_2, and the extra equation (32). The money-income ratio is given by $m(r) = \tilde{m}(r)/Y$.[40]

We use Greene and Schuh (2017) to parameterize the parameter α_1, for the weight on cash goods. Greene and Schuh provide a detailed description of the payment behavior of

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across agents.

[40]As before, $Y = Y_0 K^\theta H^{1-\theta}$, $Y_0 = 1$. If seigniorage is defined with inflation, the last term of (33) changes to $(r - \rho)\tilde{m}(r)$.

agents with daily records of U.S. consumers' spending. According Greene and Schuh, about 86 percent of spending is made using paper cash, check and other paper instruments (such as money orders), debit cards, gift cards and electronic payments.[41] As α_1 is approximately equal to the share of spending in cash goods over total consumption, this evidence implies $\alpha_1 = 0.86$. If we exclude total payments in electronic form and consider only paper instruments and debit cards, the share of spending of cash goods decreases to 47 percent, which implies $\alpha_1 = 0.47$. According to the data, therefore, $\alpha_1 = 0.47$ can be considered a lower bound for the weight on cash goods (for comparison, Cooley and Hansen 1992 consider $\alpha_1 = 0.84$).

We proceed as in section 4 to obtain the other parameters of the model. Government spending is set so that the ratio of government spending to output is equal to 20 percent. We focus on the case of seigniorage calculated as $S = rM/P$ and government spending in the form of transfers.

Figure 8 shows the money-income ratio implied by the model with endogenous and fixed N for various values of the weight on cash goods, α_1. The introduction of credit goods in fact increases the elasticity of the demand for money for fixed N. However, the effect is small. With $\alpha_1 = 0.86$ or 0.47, the money-income ratio is close to a straight line. The elasticity of the demand for money assumes larger values only for values of α_1 implausibly small.[42] On the other hand, the demand for money with endogenous N changes little for different values of α_1. The demand for money given by the model with endogenous N is robust to variations in the relative weight between cash and credit goods.

Table 4 shows the results of the model with cash and credit goods for the welfare cost and other variables. The welfare cost is calculated with (24). The benchmark economy has $r = 3\%$ p.a. (zero inflation). This benchmark economy is compared with an economy with a different value for the nominal interest rate and new equilibrium values for inflation ($\pi = r - \rho$), consumption, demand for money, hours of work and so on following equations (29)-(34). Revenues from seigniorage are used to decrease labor income taxes so that the government budget constraint holds.[43]

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[41] Tables 1b and 2b of Greene and Schuh (2017). We took the average of 2015 and 2016.

[42] As shown below, the small values of α_1 also imply implausible values for the variation of consumption within holding periods and for N.

[43] The value of N for the case with fixed holding periods is the optimal value of N at the geometric mean of interest rates in the data.

Table 4: Welfare cost of inflation and other key values

π	r	α_1 (weight on cash goods)							
		1		0.86		0.47		0.10	
		Endog.	Fixed	Endog.	Fixed	Endog.	Fixed	Endog.	Fixed
Welfare cost of inflation, relative to zero inflation (% of income)									
0	3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	5	0.14	−0.16	0.14	−0.16	0.15	−0.14	0.24	0.03
4	7	0.26	−0.32	0.26	−0.31	0.28	−0.27	0.46	0.15
10	13	0.55	−0.74	0.56	−0.71	0.61	−0.55	1.03	0.93
Holding period, N (days)									
0	3	264	264	339	308	631	572	3471	3134
2	5	225	264	262	308	487	572	2653	3134
4	7	189	264	221	308	410	572	2225	3134
10	13	138	264	161	308	299	572	1613	3134
Variation of consumption within holding periods, $c^+(T_j)/c^-(T_j)$									
0	3	1.02	1.02	1.03	1.03	1.05	1.05	1.33	1.29
2	5	1.03	1.04	1.04	1.04	1.07	1.08	1.44	1.54
4	7	1.04	1.05	1.04	1.06	1.08	1.12	1.53	1.82
10	13	1.05	1.10	1.06	1.12	1.11	1.23	1.78	3.05
Demand for money, m (relative to $r = 3\%$ p.a.)									
0	3	1.10	1.00	1.10	1.00	1.10	1.00	1.09	1.00
2	5	0.85	1.00	0.85	1.00	0.85	1.00	0.83	0.95
4	7	0.72	1.00	0.72	1.00	0.71	0.99	0.69	0.91
10	13	0.52	1.01	0.52	1.01	0.52	0.98	0.48	0.79

Welfare cost of inflation for different values of inflation. Welfare cost relative to zero inflation ($r = 3\%$ p.a.). Inflation π and nominal interest r in percent p.a. Endog. and Fixed refer to the models with endogenous and fixed N. $\alpha_1 = 1$: cash goods only. An increase in inflation is compensated with a decrease in labor income taxes, keeping the government budget constraint balanced. Government spending in the form of transfers. Transfers calibrated at 20% of GDP. Seigniorage $S = r \times M/P$. The holding period for fixed N is kept at its optimal value at $r = 3.64\%$ p.a. (the average interest rate in the data). Negative values imply a welfare gain.
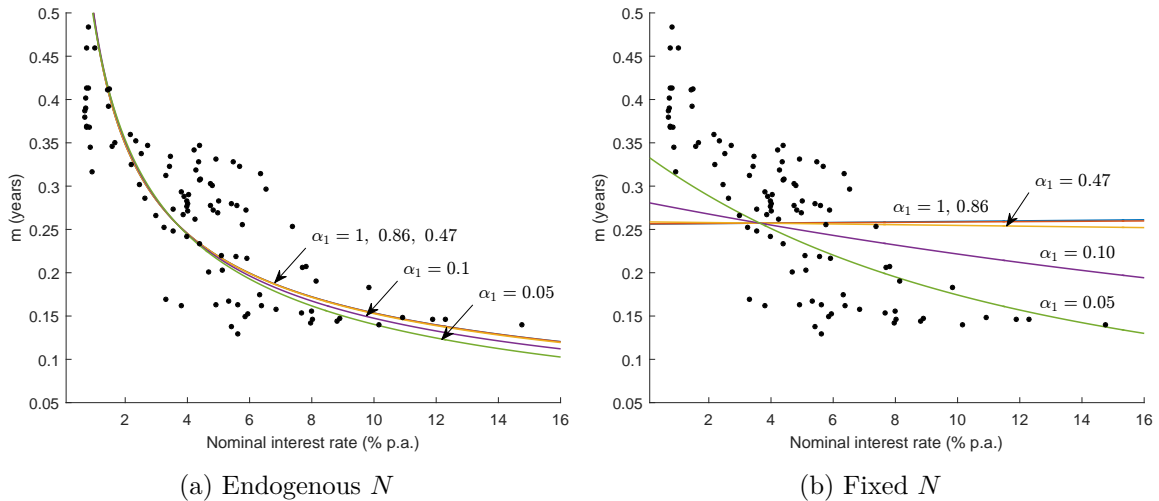
(a) Endogenous N (b) Fixed N

Figure 8: Money-income ratio implied by the model with endogenous or holding periods (N) and U.S. annual data. Endogenous N implies endogenous financial sector size. M1 for the monetary aggregate and commercial paper rate for the nominal interest rate. α_1: weight on cash goods. $\alpha_1 = 1$ implies the model with cash-goods only. The model with fixed N only implies considerable elasticity if the cash goods parameter is close to zero, whereas the data indicate $\alpha_1 > 0.4$. The model with endogenous N is robust to changes in α_1.

According to table 4, introducing credit goods changes little the conclusions on the effects of financing the government with inflation or distortionary taxes. With fixed holding periods, it is still optimal to increase inflation and decrease labor taxes for $\alpha_1 = 0.86$ and 0.47; the welfare cost is negative. The differences in predictions for the welfare cost for inflation equal to 10% p.a. between the models with endogenous or fixed N are equal to $1.26, 1.27$, and 1.16 percent of income respectively for the cases with $\alpha_1 = 1$, 0.86, and 0.47. Only with $\alpha_1 = 0.10$, the model with fixed N implies a positive welfare cost of replacing labor taxes with an inflation rate of 10% p.a. Even for this case, the welfare cost with N endogenous is higher. Moreover, $\alpha_1 = 0.10$ is much smaller than the empirical evidence of the share of cash goods.

A small value for α_1 also generates implausible values for N and for the behavior of consumption within holding periods. With $\alpha_1 = 1$ (cash goods only), the value of N under 2% p.a. inflation is equal to 225 days. This value has the same magnitude of previous estimates for similar models, as discussed above and in, for example, Alvarez et al. (2009). However, for $\alpha_1 = 0.10$, the value of N increases to 3134 days (8.6 years).[44]

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[44]The value of N increases because α_1 small implies that agents use little cash. The value of N therefore has to be adjusted to match the data for the demand for money shown in figures 2 and 8.

Small α_1 also implies a large variation of consumption of cash goods within holding periods. Agents change the behavior of cash goods consumption within holding periods to economize on cash. Consumption is high at the beginning of holding periods, at $t = T_j$, $j = 1, 2, \ldots$, and low at the end of holding periods. Call these two values $c^+(T_j)$ and $c^-(T_j)$, respectively (in the steady state, $c^-(T_j) = c^-(T_{j+1})$). The ratio c^+/c^- increases with inflation for N fixed or endogenous. However, as shown in table 4, this ratio increases much more with N fixed, as agents cannot change the frequency of exchanges from bonds to money in this case.

Although the model with credit goods increases the elasticity of the demand for money, the introduction of credit goods does not change our conclusions. The model with fixed holding periods underestimates the effects of long-run inflation. This type of model implies that it is optimal to decrease labor taxes and to finance the government with an increase in inflation.[45]

A standard cash-in-advance model does not take into account properly a change in behavior toward the use of bonds and money. Even the inclusion of the choice between cash and credit goods can only correct partially for the lack of elasticity. This correction is not enough to generate higher welfare cost.

To study the effects of long run inflation, it is necessary to include into the model a mechanism that relates the increase of inflation with the additional costs of decreasing the demand for money. If this mechanism is absent, the model will underestimate the effects of inflation.[46]

In our model, this mechanism is represented by the frequency of the conversion between bonds into money. This frequency increases as the holding period N decreases. However, it is only possible to decrease N by incurring the payment of the cost parameter more frequently. The aggregate increase in the payment of the cost parameter is interpreted as the increase in the financial sector.

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[45] As stated above, a model with fixed holding periods can only generate higher welfare cost with implausible parameters or with implausible predictions about the size of N or the variation of consumption of cash goods.

[46] Higher inflation must imply a different behavior of agents toward the financial sector. In our model with N endogenous, this is obtained by an increase in the frequency of transactions. In the cash-in-advance model with cash and credit goods, this could be obtained by a varying parameter of cash goods. Under higher inflation, agents would be willing to incur a cost to decrease α_1, as the weight of cash goods in the utility should decrease with inflation. The cost to decrease α_1 would proxy for the costs to create financial innovations to allow the payment of more goods with credit.

6   Conclusions

We analyze the effects of an increase in government spending. We consider that the increase in government spending can take the form of transfers or government consumption and that it can be financed through taxes or through inflation. The novelty is that the economy reacts to an increase in inflation by increasing the frequency of financial trades and the size of the financial sector. The frequency of financial trades is fixed in standard cash-in-advance models.

The welfare cost of financing the government with inflation increases substantially when the frequency of financial trades is taken into account. For conventional parameters, a standard cash-in-advance model underestimates the welfare cost of inflation and implies that financing with inflation is optimal. We reverse this result when we take into account the use of financial services. Financing with taxes becomes optimal.

When the government is financed with taxes, the models with fixed or endogenous holding periods imply similar predictions. This is so because the use of financial services and the demand for money do not change, as inflation does not increase, when the government is financed through taxes. The predictions differ when the change in policy involves an increase in inflation. Higher inflation increases the use of financial services. The increase in the use of financial services appears in the model as an increase in the frequency of payments to transform bonds into money. The welfare cost of inflation increases substantially.

The effects on welfare, output and consumption change. The match between data and model predictions on the demand for money improves. The increase in the use of financial services implied by the model agrees with data. We conclude that it is important to consider the use of financial services to analyze policies that imply changes in inflation.

A Appendix

A.1 Optimal Holding Period N (equation 17)

The first order conditions with respect to $T_j(n)$, $j = 2, 3, \ldots$ imply

$$
\begin{aligned}
\frac{e^{-\rho T_j}}{P(T_j)Q(T_j)}[\log c^-(T_j) - \log c^+(T_j)] = \lambda \Bigg\{ &-r(T_j)\int_{T_j}^{T_{j+1}} \frac{P(t)c(t)}{P(T_j)}dt - c^+(T_j) \\
&+ \frac{Q(T_{j-1})c^-(T_j)}{Q(T_j)} - \Gamma\left[r(T_j) - \pi(T_j)\right] \Bigg\}.
\end{aligned}
\tag{A.1}
$$

The first order conditions with respect to $c(t, n)$ imply

$$
e^{-\rho T_j}c^+(T_j)^{-1} = \lambda Q(T_j)P(T_j),
\tag{A.2}
$$

$$
e^{-\rho T_j}c^-(T_j)^{-1} = \lambda Q(T_{j-1})P(T_j).
$$

Therefore, $\log \frac{c^-(T_j)}{c^+(T_j)} = -rN_j$. The first order conditions with respect to $T_j(n)$ simplify to

$$
c^+(T_j)rN_j - r\int_{T_j}^{T_{j+1}} \frac{P(t)c(t)}{P(T_j)}dt = \Gamma\left(r - \pi\right).
\tag{A.3}
$$

In the steady state, $c(t) = c_0 e^{-r(t-T_j)}$, $c^+(T_j) = c_0$, and $c^-(T_j) = c_0 e^{-r(T_j - T_{j-1})} = c_0 e^{-rN_j}$. These equations imply

$$
c_0 r N_j - c_0 r N_{j+1}\frac{1 - e^{-\rho N_{j+1}}}{\rho N_{j+1}} = \left(r - \pi\right)\Gamma.
\tag{A.4}
$$

Setting $N_{j+1} = N_j$, $r - \pi = \rho$, and rearranging yields equation (17) in the text.

A.2 Real Money Holdings (equation 18)

Aggregate money demand is given by $M(t) = \frac{1}{N}\int_0^{t-jN} M^1(t, n)dn + \frac{1}{N}\int_{t-jN}^N M^2(t, n)dn$, $t > jN$, where $M^1(t, n)$ and $M^2(t, n)$ denote, respectively, money demand of agents in their holding periods $j + 1$ and j. The transfer time T_j of agent n is $T_j \equiv n + (j - 1)N$, $j = 1, 2, \ldots$ We have $M^1(t, n) = \int_t^{T_{j+2}} P(t)c(t)dt$, $n \in [0, t - jN)$, $T_{j+1} \leq t < T_{j+2}$, where $c(t, n) = c_0 e^{-r(t-T_{j+1})}$. In the steady state, $P(t) = P_0 e^{\pi t}$. Therefore, $M^1(t, n) = \int_t^{T_{j+2}} P_0 c_0 e^{\pi t} e^{-r(t-T_{j+1})}dt$. Analogously, $M^2(t, n) = \int_t^{T_{j+1}} P(t)c_0 e^{-r(t-T_j)}dt$, $n \in [t - jN, N]$.

Therefore, aggregate money demand is given by

$$M(t) = \frac{1}{N} \int_0^{t-jN} P_0 c_0 e^{rT_{j+1}} \frac{e^{-(r-\pi)t} - e^{-(r-\pi)T_{j+2}}}{r-\pi} dn \tag{A.5}$$
$$+ \frac{1}{N} \int_{t-jN}^N P_0 c_0 e^{rT_j} \frac{e^{-(r-\pi)t} - e^{-(r-\pi)T_{j+1}}}{r-\pi} dn.$$

This expression yields

$$M(t) = \frac{P_0 c_0 e^{rt} e^{-(r-\pi)t}}{N} \int_0^N e^{-rx} \frac{1 - e^{(r-\pi)(x-N)}}{r-\pi} dx. \tag{A.6}$$

Setting $r - \pi = \rho$ and dividing by $P(t)$ yields equation (18).

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